Physics III
ISI B.Math
Mid Semestral Exam : September 13, 2005

Total Marks: 100
Answer all questions.

1. (i) The vector potential $\mathbf{A}$ of a magnetic dipole of dipole moment $\mathbf{m}$ is given by (8)
$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\left(\frac{\mathbf{m}\times\mathbf{r}}{\mathbf{r}^3}\right)$
Show that the magnetic induction $\mathbf{B} = \nabla \times \mathbf{A}$ is given by

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{3\hat{\mathbf{r}}(\hat{\mathbf{r}}\cdot\mathbf{m}) - \mathbf{m}}{r^3}$$

(ii) In a (nonrotating) isolated mass such as a star, the condition for equilibrium is (5)

$$\nabla P + \rho\nabla\phi = 0$$

Here P is the total pressure, ρ the density, and ϕ the gravitational potential. Show that at any given point the normals to the surfaces of constant pressure and constant gravitational potential are parallel.

2. A static charge distribution produces a radial electric field

$$\mathbf{E} = A\frac{e^{-br}}{r}\hat{\mathbf{r}}$$

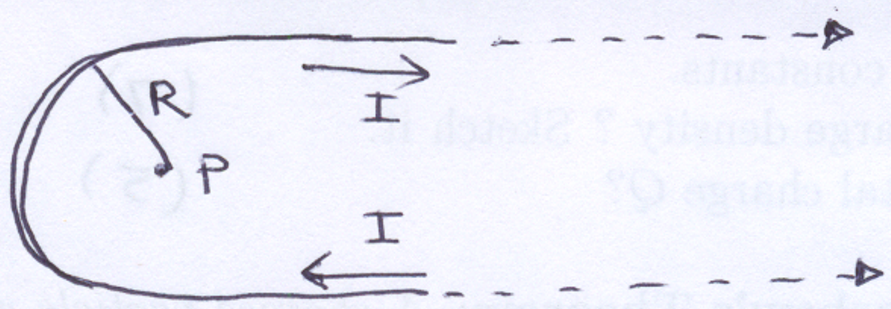
where A and b are constants.
(a) What is the charge density ? Sketch it. (7)
(b) What is the total charge Q? (5)

3. (a) Justify **Earnshaw's Theorem**: *A charged particle cannot be held in a stable equilibrium by electrostatic forces alone.* (6)
(b) Two uniform infinite sheets of electric charge densities $+\sigma$ and $-\sigma$ intersect at right angles. Find the magnitude and direction of the electric field everywhere and sketch the lines of $\mathbf{E}$. (7)
(c) Find the total energy of a uniformly charged spherical shell of total charge q and radius R. (7)

4. Charges $+q$, $-q$ lie at the points $(x, y, z) = (a, 0, a), (-a, 0, a)$ above a grounded conducting plane at $z = 0$. Find
(a) The total force on the charge $+q$. (5)
(b) The work done against the electrostatic forces in assembling this system of charges. (5)
(c) The surface charge density at the point $(a, 0, 0)$. (5)

5. (a) Find the potential $V(r, \theta)$ and the corresponding electric field for an electric dipole of dipole moment $\mathbf{p}$. Choose the dipole to be at the origin with its dipole moment pointing in the z direction. (6)
(b) Find the bound charges and the electric field produced by a uniformly polarized sphere (with polarization $\mathbf{P}$) of radius R. [*Hint: model the uniformly polarized sphere as two uniformly charged spheres – one positively charged and one negatively charged superposed with a slight displacement of their centres. Justify this model in a few lines before you proceed.*] (8)
(c) A metal sphere of radius a carries a charge Q. It is surrounded, out to radius b by a linear dielectric material of permittivity ϵ. Find the potential at the centre (relative to infinity). (6)

6. (a) Show that kinetic energy of a charged particle is conserved when it is moving in a pure magnetic field. (3)



(b) Find the magnetic field at point P for the steady current configuration shown in the figure (7)
(c) Is Ampere's law consistent with the general rule that divergence of a curl is always zero ? Show that Ampere's Law cannot be valid outside magnetostatics. (3)
(d) Show that the magnetic vector potential $\mathbf{A}$ corresponding to a magnetic field $\mathbf{B}$ can always be made to satisfy the following equation by exploiting the freedom in defining the vector potential appropriately. (7)

$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$$

where $\mathbf{J}$ is a steady current density.